

## Thursday 27 October 2022

12.00 – 13:15 *Arrival Refreshments & Registration (The Link Hotel Loughborough)*

12.00 – 13:00 *Lunch*

13.15-13.30 Welcome – A. **Veselov** and E. **Renzi**

Session 1 – Venue: Link Hotel.

Chair: E. **Parau**.

13.30-14.00 Ricardo **Barros** (Loughborough) Conjugate states and wavefronts in multi-layer fluids

14.00-14.30 Miguel **Bustamante** (UCD) Five-wave resonances in deep water gravity waves: Integrability, numerical simulations and experiments

14.30-15.00 Paul **Christodoulides** (CUT) Fluid flows around obstacles and boulders

15.00-15.30 Jean-Marc **Vanden-Broeck** (UCL) Waterfalls, weirs and spillways

15.30 – 16.00 *Refreshments*

16.00-16.30 John M. **Dudley** (Université de Franche Comté, **online**) Rogue waves and analogies in optics and oceanography

## Friday 28 October 2022

7.00 – 9.00 *Breakfast (speakers only)*

9.00 – 9.30 *Registration (The Link Hotel Loughborough)*

Session 2 – Venue: Link Hotel.

Chair: P. **Milewski**.

9.30-10.00 Tom **Bridges** (Surrey) Superharmonic instability and deep water wave breaking

10.00-10.30 Philippe **Guyenne** (Delaware) Hamiltonian Dysthe equation for deep-water gravity waves

10.30 – 11.00 *Refreshments*

Session 3 – Venue: Link Hotel.

Chair: T. **Bridges**

11.00-11.30 Paul **Milewski** (Bath) True mode-2 solitary waves in 3-layer flows

11.30-12.00 Gerard **Iooss** (Nice, IUF) Patterns and quasipatterns from the superposition of two hexagonal lattices

12.00-12.30 Emilian **Parau** (UEA) Waves under ice sheets

12.30 – 13.30 *Lunch & Registration*

Session 4 – Venue: Link Hotel.  
Chair: P. **Guyenne**

- 13.30-14.00 Sarah **Gallagher** (MET Éireann) Monitoring and Predicting the Irish Wave Climate: Extremes, Analysis and Continuity
- 14.00-14.30 Cathal **Cummins** (Heriot-Watt) Low-frequency hydrodynamic resonance in raft-type WECs with sloping walls
- 14.30-15.00 Emiliano **Renzi** (Loughborough) A moving particle semi-implicit numerical wave flume to model design waves
- 15.00-15.30 Nick **Trefethen** (Oxford) Sixteen examples of AAA rational approximation

*15.30 – 15.50 Refreshments*

- 15.50-16.20 Dimitra **Salmanidou** (UCL) Merging geophysics and economics for future tsunami loss in Indonesia

### **Social Events**

Public Lecture – Venue: Room 0.01 Schofield Building (Maths Department)  
Chair: C. **Linton**

- 17.30-18.30 Frederic **Dias** (ENS Paris-Saclay, UCD): **The fascination of ocean waves.**

### Banquet Dinner

19.15 **Burleigh Court Hotel** (Loughborough University)

## **Saturday 29 October 2022**

*7.00 – 9.00 Breakfast (speakers only)*

Session 5 – Venue: Link Hotel  
Chair: K. **Khusnutdinova**

- 9.00-9.30 Xiao Bo **Chen** (Bureau Veritas) Fundamental solutions of free-surface potential flows with viscous effect
- 9.30-10.00 Simone **Michele** (Plymouth) Heat transfer beneath standing and propagating waves
- 10.00-10.30 Jean-Michel **Ghidaglia** (UM6P, Benguerir, Morocco and ENS Paris-Saclay) A simulation model for a 187 km long slurry pipeline

*10.30 – 11.00 Refreshments*

Session 6 – Chair: E. **Renzi**

11.00-11.30 Nicolas **Vayatis** (ENS Paris Saclay) TBA

11.30-12.00 Karima **Khusnutdinova** (Loughborough) Internal ring waves in a three-layer fluid over a linear shear current

12.00-12.30 Discussions

12.30 – 13.30 *Lunch*

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**Abstracts**

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Frederic **Dias** (ENS Paris Saclay, UCD)

**Public Lecture: The fascination of ocean waves**

Scales in ocean waves range from the molecule size (of the order of the nanometre) to the size of the earth (several thousands of kilometres). In this talk, we will provide illustrations of this wide range of waves. We will describe the scientific challenges associated with the study of ocean waves and how a multidisciplinary approach is often the only way to move forward.

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Ricardo **Barros** (Loughborough)

**Conjugate states and wavefronts in multi-layer fluids**

Abstract: Two horizontally uniform flows are said to be conjugate if all three basic physical conservation laws of mass, momentum and energy for Euler equations hold. Many flow systems are known in which the study of conjugate states is crucial to the understanding of observed wave phenomena. Three of them will be considered in this talk: a two-layer flow (with and without a free surface) and a three-layer flow between two rigid walls. We will show that, for each one of these systems, strongly nonlinear theory initially developed by Miyata (1988), Choi and Camassa (1999) is able to capture the exact same conjugate states of the fully nonlinear equations. However, the existence of conjugate states does not always imply the existence of a wavefront connecting the two uniform flows. This is one of the aspects we will explore, together with their relation with limiting solitary waves for the systems considered. This is joint work with Paul Milewski and Alex Doak.

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Tom **Bridges** (Surrey)

**Superharmonic instability and deep water wave breaking**

The superharmonic (SH) instability is one of the two principal instabilities of nonlinear Stokes waves, the other being the Benjamin–Feir instability. The SH instability differs from the Benjamin–Feir instability in that the SH perturbation has the same wavelength as the

basic periodic travelling wave, and arises at finite, and sometimes very large, amplitude. A seminal observation of Tanaka, Dold, Lewy, Peregrine in 1987 was the emergence of wave breaking from an SH instability. This observation was expanded and generalised by Jillians, Longuet-Higgins, Dörmmermuth, Holyer, and others. However all this work is in 2D. In this talk I discuss the extension of these ideas to 3D, focussing on the SH instability of short-crested Stokes waves (SCWs). The SH instability for a range of SCW parameters is computed as well as the eigenfunctions. The eigenfunctions are showing a 3D variant of the dipole structure that is important for generating wave breaking. Addition of a multiple of the eigenfunction to the basic SCW shows a steepening of the overall wave. To prove the time dependent wave breaking, simulation of the time dependent problem is in progress. This talk is based on joint work with Frederic Dias and Matt Turner.

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Miguel **Bustamante** (UCD Dublin)

### **Five-wave resonances in deep water gravity waves: Integrability, numerical simulations and experiments**

In this work we consider the problem of finding the simplest arrangement of resonant deep-water gravity waves in one-dimensional propagation, from three perspectives: Theoretical, numerical and experimental. Theoretically this requires using a normal-form Hamiltonian that deals with 5-wave resonances. The simplest arrangement is based on a triad of wavevectors  $K_1 + K_2 = K_3$  (satisfying specific ratios) along with their negatives, corresponding to a scenario of encountering wavepackets, amenable to experiments and numerical simulations. The normal-form equations for these encountering waves in resonance are shown to be non-integrable. Numerical simulations of the governing equations in natural variables using pseudospectral methods require the inclusion of up to 6-wave interactions, which imposes a strong dealiasing cut-off in order to properly resolve the evolving waves. We study the resonance numerically by looking at a target mode in the base triad and showing that the energy transfer to this mode is more efficient when the system is close to satisfying the resonant conditions. We first look at encountering plane waves with base frequencies in the range 1.32 -- 2.35 Hz and steepnesses below 0.1, and show that the time evolution of the target mode's energy is dramatically changed at the resonance. We then look at a scenario that is closer to experiments: Encountering wavepackets in a 400m long numerical tank, where the interaction time is reduced with respect to the plane-wave case but the resonance is still observed; by mimicking a probe measurement of surface elevation we obtain efficiencies of up to 10% in frequency space after including near-resonant contributions. Finally, we perform preliminary experiments of encountering wavepackets in a 35m long tank, which seem to show that the resonance exists physically. The measured efficiencies via probe measurements of surface elevation are relatively small, indicating that a finer search is needed along with longer wave flumes with much larger amplitudes and lower frequency waves.

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Xiaobo **Chen** (Bureau Veritas, Saint-Herblain, France)

### **Fundamental solutions of free-surface potential flows with viscous effect**

Following the work in Dias *et al.* (2008, *Physics Letter*), water flows beneath the free surface described by the linear incompressible Navier-Stokes equations and appropriate boundary conditions are represented by the sum of an irrotational and a solenoid vector fields according to Helmholtz decomposition. Through the joint Fourier-Laplace transform, an important relationship between the vertical component of rotational flow velocity and horizontal variations of free-surface elevations is found and used to derive a new set of kinematic and dynamic boundary conditions on the free surface satisfied by the velocity potential with the viscous effect. Application of new boundary conditions to the flow generated by a pulsating and translating source leads to the free-surface Green function with viscous effect.

By applying the same method based on Helmholtz decomposition and the joint Fourier-Laplace transform, the direct analysis on the time-harmonic Oseen equations associated with a pulsating and translating source gives the same fundamental solution, i.e. the free-surface Green function with viscous effect. It is shown that the rotational part of water flows is proportional to the viscous coefficient as the leading order so that the flow is well dominated by the potential part. Unlike the classical inviscid Green function which is highly oscillatory and singular when both source and field points approach to the free surface, shown in Chen & Wu (2001, *JFM*), the free-surface Green function with viscous effect represents properly wavy properties of smooth magnitudes which are physically acceptable. Indeed, the application of the Green function with viscous effect in the seakeeping problem with forward speed shows its soundness and produces excellent results comparing to physical measurements in ship model tests.

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Paul **Christodoulides** (Cyprus University of Technology)

### **Fluid flows around obstacles and boulders**

Fluid flows around obstacles constitute a classic problem in fluid dynamics. Such obstacles may be convex-shaped protrusions of the bottom of some flat rigid surface, over which some fluid flows. Steady two-dimensional fluid flows over an obstacle can be solved using complex variable methods. In particular, we study the impact of a flow hitting a vertical wall of finite extent. The fluid overtops the finite vertical wall as shown in Figure 1(a). Here we consider free-surface flows past a semi-infinite step at the bottom of a channel for an inviscid and incompressible fluid; the flow is steady and irrotational. The flows is uniform far upstream with constant velocity and constant depth, while far downstream the flow is also uniform with a different constant velocity and constant depth. The solution of such problems depends on the depth ratios and on the dimensionless upstream and downstream Froude numbers. We will present the numerical procedure with various solutions of the problem, including limiting flows. Relevant pressure and forces will be also addressed with the discussion touching the “boulder” problem.

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Cathal Cummins (Heriot-Watt University)

### **Low-frequency hydrodynamic resonance in raft-type WECs with sloping walls**

Attenuator Wave Energy Converters (WECs) typically consist of two or more sections, hinged at a central location, allowing the device to convert energy from the relative pitching motion of the sections. Such WECs typically have device lengths that are longer than the wavelength of the incident waves: e.g., Pelamis's P2 WEC. Mocean Energy's Blue X device (deployed at EMEC) resembles an attenuator, but its design includes sloping walls on the front and rear of the device that substantially enhance the device's wave power capture (compared with a simple hinged raft). This enhancement occurs at multiple incident wavelengths that are longer than the device's length, yet a satisfying description of the hydrodynamics driving this enhancement has not been previously reported. In this talk, we examine the fluid-mechanics resonance that permits this tuning to take place and we argue that this low-frequency resonance closely resembles Helmholtz resonance [1].

[1] Cummins CP, Scarlett GT, Windt C. Numerical analysis of wave-structure interaction of regular waves with surface-piercing inclined plates. **J Ocean Eng. Mar Energy**. 2022;8(1):99–115.

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John M. Dudley (Université de Franche Comté)

### **Rogue waves and analogies in optics and oceanography**

In 2007, experiments in optics suggested that instabilities observed in the nonlinear propagation of ultrashort pulses in optical fibre could be analogous to hydrodynamic rogue waves. This proposal was greeted with great interest in both the optics and hydrodynamics communities, but it soon became apparent that establishing such an analogy rigorously would require a great deal of additional work. Moreover, at first glance, the optics experiments seemed so complex that it was difficult to know where to start. It is here where our joint work with Frederic Dias proved especially fruitful in identifying specific optical systems where such an analogy was really valid. Over 12 years of wonderful collaboration has resulted in a number of extremely significant results such as the first observation of the Peregrine soliton, as well as experiments on many other novel classes of breather solution. This talk will provide an overview of the field of optical rogue waves, highlighting the many areas where Frederic's input has been vital in developing our understanding of the underlying physics.

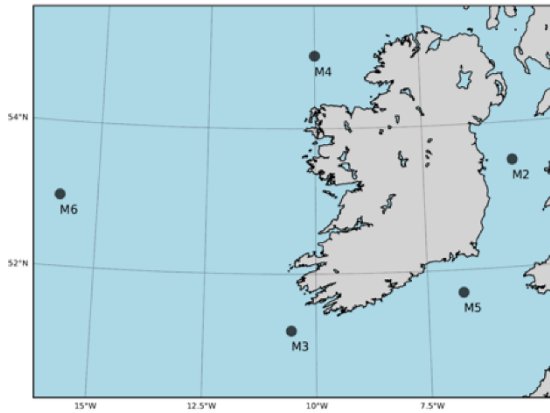
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Sarah Gallagher (MET Eireann)

### **Monitoring and Predicting the Irish Wave Climate: Extremes, Analysis and Continuity.**

On the 28th October 2020, the M6 weather buoy off the West coast of Ireland recorded a wave of 32.3m height in a sea state of significant wave height,  $H_{m0}$ , ~14m. This represents a new record maximum wave height, for the M6 buoy, the Irish Marine Data Buoy Observation Network (IMDBON) and is amongst the largest individual waves recorded. In this presentation a brief history of observed wave heights, climatology's and future predictions for Irish waters will be shown, along with recent changes in measurement techniques for the IMDBON network. Future research and development plans will also be discussed, including

the development of a national coastal flood forecasting service, adapting to the future climate and the challenges these present for Ireland in the ever-energetic wave climate of the North Atlantic.



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Jean-Michel **Ghidaglia** (MSDA, UM6P, Benguerir, Morocco & Centre Borelli, ENS Paris-Saclay, France)

### **A simulation model for a 187 km long slurry pipeline**

OCP (the world's largest producer of phosphate and phosphate-based products) operates since 2014 a 187 km long pipeline in Morocco to transport phosphate ore from the Khourigba mine to the Atlantic Ocean in Jorf Lasfar. The fluid is a slurry obtained by mixing fresh water and phosphate ores previously reduced to powder.

Our team at MSDA has delivered this September to OCP a simulator of the slurry pipe that can be used by the operators of the installation in order to improve the operations. The model is able to simulate one month of physical time in 20 minutes on a Laptop.

The purpose of this talk is to present and discuss the simulation model used. Some results with actual data are also presented.

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Philippe **Guyenne** (Delaware University)

### **Hamiltonian Dysthe equation for deep-water gravity waves**

A new Hamiltonian version of Dysthe's equation is derived for weakly modulated gravity waves on deep water. A key ingredient in this derivation is a Birkhoff normal form transformation that eliminates all non-resonant cubic terms and allows for a refined reconstruction of the free surface. This modulational approximation is tested against numerical solutions of the classical Dysthe's equation and against direct numerical simulations of Euler's equations for nonlinear water waves. Very good agreement is found in the context of Benjamin–Feir instability of Stokes waves, for which an analysis is provided. An extension of our Hamiltonian model incorporating exact linear dispersion as well as an alternate spatial form are also proposed. Both the 2D and 3D problems are considered. Comparison with laboratory experiments is also shown.

Gerard **Iooss** (LJAD Nice and IUF)

### **Patterns and quasipatterns from the superposition of two hexagonal lattices**

We consider the Swift - Hohenberg PDE with quadratic as well as cubic nonlinearities, and look for solutions built on a lattice which is the superposition of two hexagonal lattices rotated by an angle  $\beta$  with respect to each other. We prove existence of several new types of quasipatterns, in particular quasipatterns made from the superposition of hexagons and stripes (rolls) oriented in almost any direction and with any relative translation, and quasipatterns made from the superposition of hexagons with unequal amplitude (provided the coefficient of the quadratic nonlinearity is small). We consider the periodic case as well, and extend the class of known solutions, including the superposition of hexagons and stripes. For the quasiperiodic cases, the proofs follow the process used by the author with B.Braaksma and L.Stolovitch on a simpler problem. This a collaboration with A. M. Rucklidge (Leeds)

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Karima **Khusnutdinova** (Loughborough University)

### **Internal ring waves in a three-layer fluid over a linear shear current**

Oceanic internal waves often have curvilinear fronts and propagate over various currents. We present the first study of long weakly-nonlinear internal ring waves in a three-layer fluid in the presence of a background linear shear current. The leading order of this theory leads to the angular adjustment equation - a nonlinear first-order differential equation describing the dependence of the linear long-wave speed on the angle to the direction of the current. Ring waves correspond to singular solution (envelope of the general solution) of this equation, and they can exist only under certain conditions. The constructed solutions reveal qualitative differences in the shapes of the wavefronts of the two baroclinic modes: the wavefront of the faster mode is elongated in the direction of the current, while the wavefront of the slower mode is squeezed. Moreover, different regimes are identified according to the vorticity strength. When the vorticity is weak, part of the wavefront is able to propagate upstream. However, when the vorticity is strong enough, the whole wavefront propagates downstream. A richer behaviour can be observed for the slower mode. As the vorticity increases, singularities of the swallowtail-type may arise and, eventually, solutions with compact wavefronts crossing the downstream axis cease to exist. We show that the latter is related to the long-wave instability of the base flow. We obtain analytical expressions for the coefficients of the cKdV-type amplitude equation, and numerically model the evolution of the waves for both modes. The initial evolution is in agreement with the leading-order predictions for the deformations of the wavefronts. Then, as the wavefronts expand, strong dispersive effects in the upstream direction are revealed. Moreover, when nonlinearity is enhanced, fission of waves can occur in the upstream part of the ring waves. Joint work with D. Tseluiko, N.S. Alharthi and R. Barros.

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Simone **Michele** (Plymouth University)

### **Heat transfer beneath standing and propagating waves**

A mathematical model is developed to investigate seabed heat transfer processes enhanced by long-crested ocean waves. The unsteady convection-diffusion equation for fluid temperature includes terms depending on the velocity field in the laminar boundary layer, which can be derived in a similar way as for mass transfer near the seabed. Here we consider propagating and reflected waves from a vertical structure, which in turn complicate the convective term in the governing equation. The roles of heat source profile, location and strength on heat flux dynamics are explained, providing insights into seabed temperature forced convection mechanisms that are enhanced by free-surface waves.

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Paul **Milewski**

### **True mode-2 solitary waves in 3-layer flows**

Stratified fluids with narrow regions of rapid density variation with respect to depth (pycnoclines) are often modelled as layered flows. In this talk we shall examine horizontally propagating internal waves of the second baroclinic mode (mode-2), by computing travelling wave solutions within three-layer fluid. We will be presenting numerical solutions to both the full Euler system and to a reduced model called the three-layer Miyata-Choi-Camassa (MCC3) equations. Mode-2 waves (typically) occur within the linear spectrum of mode-1 waves, and are hence generically associated with an infinite energy resonant mode-1 oscillatory tail. However, in line with recent results for the MCC3 system by Barros, Choi & Milewski (JFM, 2020), we will present numerical evidence that these oscillations can be found to have zero amplitude, resulting in families of truly localised solutions (so called embedded solitary waves). We also discuss limiting solutions which are related to heteroclinic connections between the so called conjugate states of the system (which coincide for Euler and MCC3). This is joint work with Ricardo Barros and Alex Doak.

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Emilian **Parau** (University of East Anglia)

### **Waves under floating ice sheets**

A short review of the waves propagating under an ice sheet in two and three dimensions will be presented. The results will include weakly nonlinear model equations and fully-nonlinear computed solutions. The ice will be modelled as a thin elastic plate in the majority of cases, but we will also consider the attenuation of waves due to broken ice floes.

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Emiliano **Renzi** (Loughborough University)

### **A moving particle semi-implicit numerical wave flume to model design waves**

This talk will present recent developments in the numerical modelling of highly nonlinear waves in viscous liquid, using a Lagrangian approach known as Moving-Particle Semi-Implicit (MPS) method.

The talk will give a review of how MPS models have evolved in the last decade to allow reproducing water waves and will discuss existing challenges. Then, a high-order MPS model will be introduced to build a novel numerical wave flume (NWF) for modelling design waves in the context of ocean and coastal engineering applications. A key feature of the model is the implementation of a new scheme for the artificial viscosity, which is original to the Smoothed Particle Hydrodynamics (SPH) method and is here formalised into the MPS context. Finally, the talk will show validation of the NWF-MPS model with available analytical and experimental benchmarks, and discuss the model capability to reproduce nonlinear wave phenomena.

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Dimitra **Salmanidou** (UCL)

### **Merging geophysics and economics for future tsunami loss in Indonesia**

Tsunamis form a recurrent hazard for the Indonesian coastlines with a disastrous impact. Catastrophe modelling, often used by the (re-)insurance industry to estimate perils, forms an effective tool to understand the economical impact of future tsunami hazard at vulnerable coastlines. To do so, several hazard and risk components need to be integrated in the stochastic model. In this study, inundation footprints generated by earthquake tsunamis in the Java trench are computed at high resolution scales and the vulnerability with respect to livelihoods at different intensity scales of the hazard is measured to compute loss. A surrogate approach is integrated in the process to create a synthetic plethora of the events and reduce the waste of computational resources. The process and results will be discussed in my talk.

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Nick **Trefethen** (University of Oxford)

### **Sixteen example of AAA rational approximation**

For the first time, a method has become available for fast computation of near-best rational approximations on arbitrary sets in the real line or complex plane: the AAA algorithm (Nakatsukasa-Sète-T. 2018). This talk will focus not on the derivation of the algorithm but on sixteen demonstrations of the kinds of things we can do, all across applied mathematics, with a black-box rational approximation tool.

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Jean-Marc **Vanden-Broeck** (UCL)

### **Waterfalls, weirs and spillways**

Two-dimensional free surface potential flows are considered. The effects of gravity are included in the dynamic boundary condition. It is assumed that the flow configuration involves falling jets. Examples involve waterfalls and weir flows. The fully nonlinear problems are solved numerically by series truncation. Early work on the subject is first reviewed, including joint work with Frédéric Dias. Recent results will then be presented. These new results confirm the early work and extend it.